

# FORECASTING PUBLIC EXPENDITURE BY USING LINEAR AND NON-LINEAR MODELS

**Cagdas Hakan Aladag**

Hacettepe University, Faculty of Science,  
Department of Statistics, Turkey

*chaladag@gmail.com*

## ABSTRACT

Public expenditure forecasting is an important task which attracts attention from many researchers. This forecasting problem is crucial for the success of future public financial management or budget management approaches in a country. The main purpose of this study is to develop some efficient forecasting models for public expenditure in order to reach high accuracy level. For this purpose, different linear and non-linear models based on autoregressive integrated moving average (ARIMA), exponential smoothing, and artificial neural networks are developed. This study applies various linear and non-linear models to the expenditures of 1980-2010 of two Turkish public institutions, namely, the State Planning Organization and the Court of Accounts to achieve accurate forecast levels. Different linear and non-linear forecasting models are applied to the related time series and obtained results are compared. As a result of the implementation, it is observed that the artificial neural networks provide very accurate public expenditure forecasts for these public institutions, suggesting that the artificial neural networks is a very effective tool for the public expenditure forecasting.

## KEY WORDS:

*ARIMA, Artificial neural networks, Forecasting, Public expenditure, Time series.*

## INTRODUCTION

The accurate forecast of public expenditure is a vital issue for many government organizations. Government organizations worldwide need accurate public expenditure forecasting in order to plan future strategies. The public expenditure forecasting can be crucial for the success of future public financial management or budget management approaches in a country (Basaran et al., 2012). There are some public expenditure studies in the literatures. Some of them are Bagdigen (2005), Abeysinghe and Jayawickrama (2008), and Tang (2009). Public expenditure data can be considered as time series. It could be daily, monthly, seasonally or yearly. These time series can be modeled by conventional linear forecasting models such

as autoregressive integrated moving average (ARIMA) or exponential smoothing models. On the other hand, it is a well-known fact that real world time series generally have both linear and non-linear structures (Yolcu et al., 2013). Therefore, non-linear forecasting approaches should also be utilized when public expenditure data is forecasted. In the literature, one of the best non-linear time series forecasting techniques is artificial neural networks method (Aladag et al., 2013).

In recent years, artificial neural networks approach has been applied to many areas; one of them is the time series forecasting (Aladag and Egrioglu, 2012). Since the artificial neural networks can model both nonlinear and linear structure of time series, using the artificial neural networks in forecasting can give more accurate results than the other methods (Aladag et al., 2009). In the literature, artificial neural networks models have proved their success in forecasting various time series from different fields (Aladag et al., 2008). Therefore, using artificial neural networks for time series forecasting have drawn a great amount of attention in recent years (Aladag et al., 2011).

In this study, it is aimed to obtain accurate forecasts for public expenditure. In order to reach high accuracy level, different linear and non-linear forecasting models were utilized. The expenditures of 1980-2010 of two Turkish public institutions which are the State Planning Organization and the Court of Accounts were used in the implementation. Conventional linear methods such as ARIMA and exponential smoothing were applied to these time series. Various artificial neural networks models, which are non-linear, were also employed to forecast these time series. All results obtained from different approaches are presented and compared with each other. According to obtained forecasting results, it was observed that the most accurate forecasts are obtained when artificial neural networks models are utilized for both time series. Although, artificial neural networks method is a very effective forecasting tool, there have not been enough public expenditure studies use this effective forecasting approach. Therefore, this study can guide future public expenditure forecasting studies.

In the next section, brief information about artificial neural networks approach is presented. Section 3 gives the implementation in details. Finally, Section 4 concludes the paper.

## **ARTIFICIAL NEURAL NETWORKS**

Aladag et al. (2008) gave brief information about the artificial neural networks as follows: ‘What is an artificial neural network?’ is the first question that should be answered. Picton (1994) answered this question by separating this question into two parts. The first part is why it is called as artificial neural network. It is called as artificial neural network because it is a network of interconnected elements. These

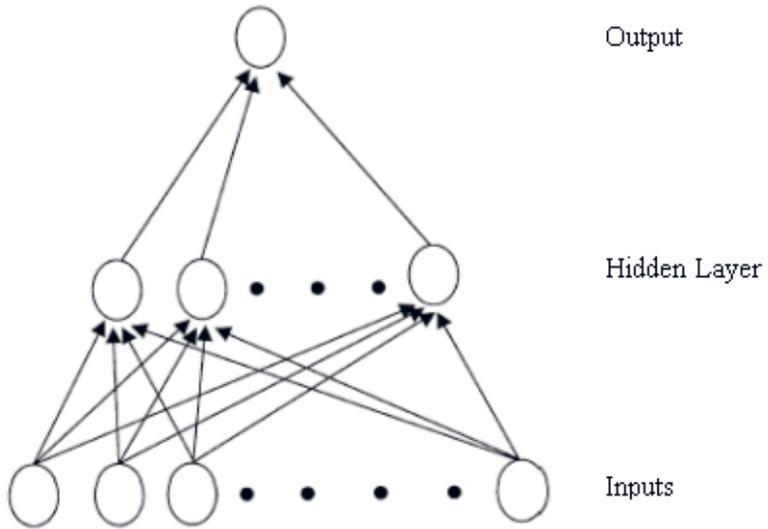
elements were inspired from studies of biological nervous systems. In other words, the artificial neural networks are an attempt at creating machines that work in a similar way to the human brain by building these machines using components that behave like biological neurons.

The second question is what an artificial neural network does? The function of an artificial neural network is to produce an output pattern when presented with an input pattern. In forecasting, the artificial neural networks are mathematical models that imitate the biological neural networks. The artificial neural networks consist of some elements. Determining the elements of the artificial neural networks issue that affect the forecasting performance of artificial neural networks should be considered carefully. Elements of the artificial neural networks are generally given as network architecture, learning algorithm and activation function (Gunay et al., 2007).

One critical decision is to determine the appropriate architecture, that is, the number of layers, the number of nodes in each layers and the number of arcs which interconnects with the nodes (Zurada, 1992). However, in the literature, there are not general rules for determining the best architecture. Therefore, several architectures should be tried for the correct results. There are various types of artificial neural networks. One of them is called as feed forward neural networks. The feed forward neural networks have been used successfully in many studies (Gunay et al., 2007). In the feed forward neural networks, there are no feedback connections. The broad feed forward neural network architecture that has single hidden layer and single output is given as an illustration in Figure 1.

Learning of an artificial neural network for a specific task is equivalent to finding the values of all weights such that the desired output is generated by the corresponding input. Various training algorithms have been used for the determination of the optimal weights values. The most popularly used training method is the back propagation algorithm presented by Smith (2002). In the back propagation algorithm, learning of the artificial neural networks consists of adjusting all weights considering the error measure between the desired output and actual output (Cichocki and Unbehauen, 1993).

Another element of the artificial neural networks is the activation function. It determines the relationship between inputs and outputs of a network. In general, the activation function introduces a degree of the non-linearity that is valuable in most of the artificial neural networks applications. The well-known activation functions are logistic, hyperbolic tangent, sine (or cosine) and the linear functions. Among them, logistic activation function is the most popular one (Zhang et al., 1998).



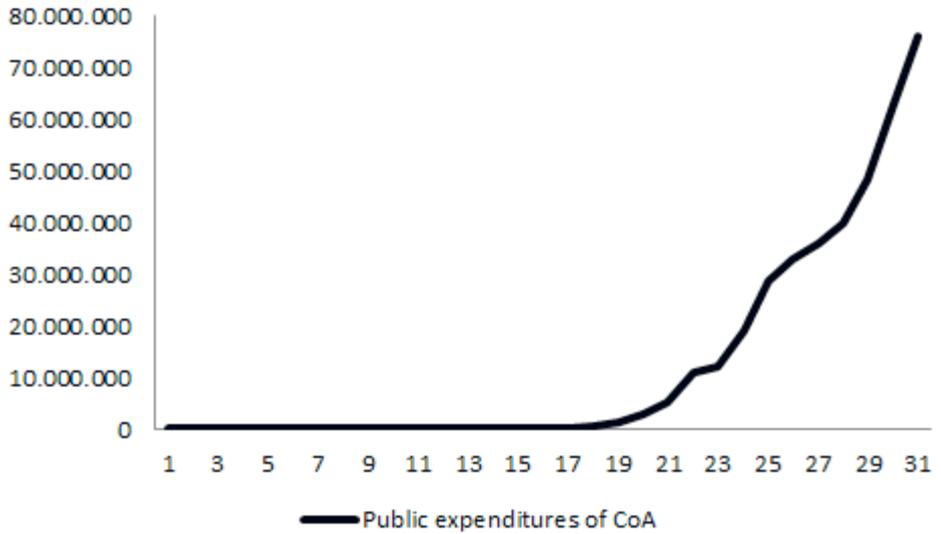
**Figure 1.** A broad feed forward neural network architecture

### THE IMPLEMENTATION

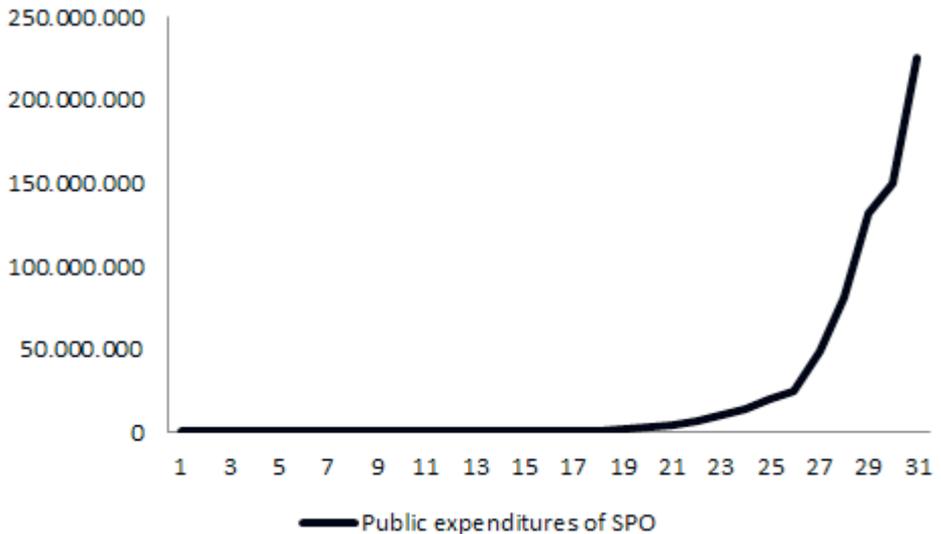
In the application, public expenditures of two Turkish public institutions, namely, State Planning Organization (SPO) and the Court of Accounts (CoA) were forecasted by using various linear and non-linear forecasting models. The graphs of these time series are presented in Figure 2 and 3, respectively. In these figures, vertical and horizontal axes represent public expenditure (Turkish Liras) and time, respectively. Each series includes annually 31 observations. The last 5 observations corresponding the years between 2007 and 2010 are used for test set and the rest of the observations are employed for training. First of all, all models are trained and model parameters are determined by using the training set. Then, the forecasting performances of the models are evaluated by using root mean square error (RMSE) which is calculated over the test set. RMSE can be calculated by using the formula given in (1).

$$RMSE = (1)$$

where  $y_p$  and  $nt$  represent crisp time series, defuzzified forecasts, and the number of forecasts, respectively.



**Figure 2.** Public expenditures of CoA (Turkish Liras)



**Figure 3.** Public expenditures of SPO (Turkish Liras)

Non-linear models such as ARIMA and exponential smoothing were applied to these time series. When ARIMA models are employed, ARIMA(0,1,0) model

was found as the best model for both time series. Brown's exponential smoothing (BES) model was utilized to forecast the time series. Also, various artificial neural networks (ANN) models, which can model both linear and non-linear part of time series, were used. Both the number of inputs and neurons in the hidden layer are changed from 1 to 12. Thus, 144 architectures are totally examined for each time series since one neuron is used in the output layer. Then, the best architecture that has the minimum RMSE value calculated over the test set is selected. In other words, the RMSE is preferred as performance measure since it is a well-known measure for forecasting. When the best architecture is being searched, the other elements of the artificial neural networks are fixed. The logistic activation function is used in all of the neurons of networks. Levenberg Marquardt algorithm is employed as training algorithm because of the high convergence speed of the algorithm.

When CoA time series was analyzed artificial neural networks, the best architecture was found as 1-3-1. It means that the best architecture has 1, 3, and 1 neurons in input, hidden and output layers, respectively. For SPO series, 5-5-1 architecture was chosen as the best architecture which gives the most accurate forecasts.

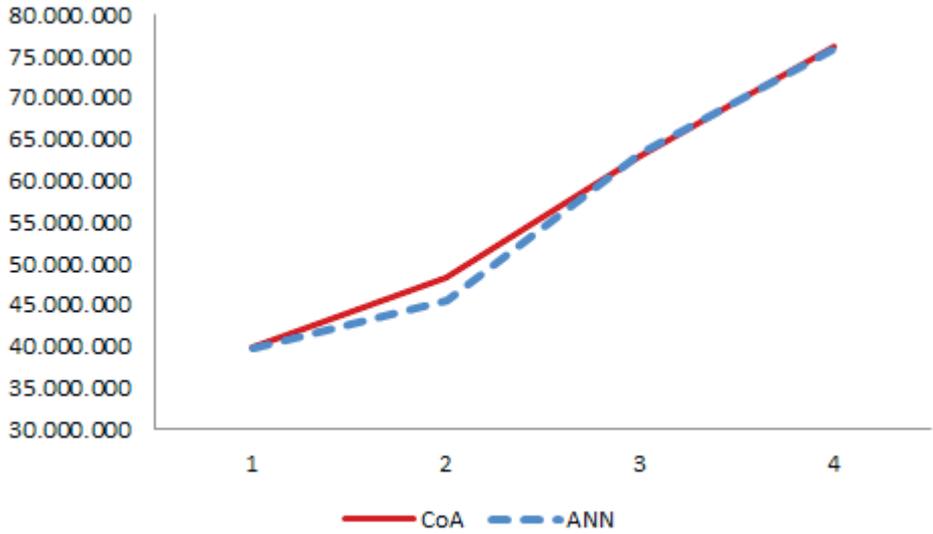
RMSE values calculated over test sets are summarized in Table 1 for all methods. In Table 1, Brown's exponential smoothing and artificial neural networks methods are represented by BES and ANN, respectively.

**Table 1.** RMSE values obtained from all forecasting approaches.

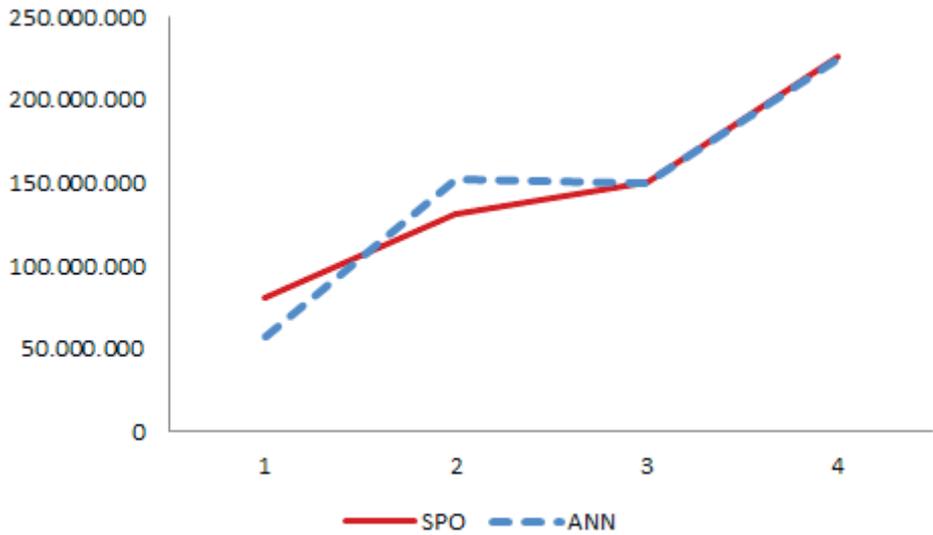
<b>Time series</b>	<b>BES</b>	<b>ARIMA</b>	<b>ANN</b>
Public expenditures of CoA	12.93x10 <sup>6</sup>	10.89x10 <sup>6</sup>	1.48x10 <sup>6</sup>
Public expenditures of SPO	53.65x10 <sup>6</sup>	49.09x10 <sup>6</sup>	15.74x10 <sup>6</sup>

According to Table 1, for both time series, it is clearly seen that the most accurate forecasts are obtained when artificial neural networks method is used in terms of RMSE criterion. For both time series, the second best method is ARIMA. These results indicate that these public expenditure series include non-linearity. Therefore, artificial neural networks models produced the best forecasts for both time series.

Forecasting performance of artificial neural networks with the best results can also be examined visually. The graphs of the forecasts obtained from the best artificial neural network models and the observed values for test sets are given in Figure 4 and 5, respectively. In these graphs, vertical and horizontal axes represent public expenditure (Turkish Liras) and time, respectively. When the graphs are examined, it is clearly seen that the calculated forecasts are very good.



**Figure 4.** The obtained forecasts and the observed values for the CoA series.



**Figure 5.** The obtained forecasts and the observed values for the SPO series.

## CONCLUSION

The accurate forecast of public expenditure is crucial for government organizations worldwide. To forecast public expenditure, some approaches can be utilized. ARIMA and exponential smoothing methods are conventional forecasting methods and based on linear models. On the other hand, since it is a well-known fact that real-world time series generally have both linear and non-linear structures, non-linear forecasting approaches such as artificial neural networks should also be used to forecast public expenditure. However, artificial neural networks method has been rarely used to forecast public expenditure although this method is a very effective forecasting tool. In this study, linear and non-linear forecasting techniques are utilized to forecast public expenditures of two Turkish public institutions, namely, State Planning Organization and the Court of Accounts. ARIMA, Brown's exponential smoothing, and artificial neural network models are applied to these time series. All obtained forecasting results are presented and compared with each other. The implementation on the expenditure data of the SPO and the CoA has proved that the artificial neural networks approach is a very useful prediction tool, which can also be employed by other public institutions to forecast their expenditures more accurately.

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